Eur. Phys. J. C 32, 29–43 (2003)
Digital Object Identifier (DOI) 10.1140/epjc/s2003-01347-5

THE EUROPEAN
PHYSICAL JOURNAL C

Weak decays of the B_c meson to B_s and B mesons in the relativistic quark model

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Received: 14 August 2003 / Revised version: 3 September 2003 /
Published online: 7 November 2003 –

Abstract. Semileptonic and non-leptonic decays of the B_c meson to B_s and B mesons, caused by the $c \to s, d$ quark transitions, are studied in the framework of the relativistic quark model. The heavy quark expansion in inverse powers of the active c and spectator $\bar{b}$ quark is used to simplify calculations while the final s and d quarks in the B_s and B mesons are treated relativistically. The decay form factors are explicitly expressed through the overlap integrals of the meson wave functions in the whole accessible kinematical range. The obtained results are compared with the predictions of other approaches.

1 Introduction

The B_c meson discovered at Fermilab [1] is the only quark–antiquark bound system ($\bar{b}c$) composed of heavy quarks (b, c) with different flavors, thus being flavor asymmetric. The investigation of the B_c meson properties (mass spectrum, decay rates, etc.) is therefore of special interest compared to symmetric heavy quarkonium ($b\bar{b}$, $c\bar{c}$) ones. The difference of quark flavors forbids the annihilation of B_c into gluons. As a result the pseudoscalar $\bar{b}c$ state is much more stable than the heavy quarkonium one and decays only weakly. It serves as a final state for the pionic and radiative decays of the excited $\bar{b}c$ states (lying below the BD threshold). Experimental study of the B_c mesons is planned both at the Tevatron and Large Hadron Collider (LHC) (for a recent review see e.g. [2] and references therein).

Since both quarks (b, c) composing the B_c meson are heavy, their weak decays contribute comparably to the total decay rate. Thus there are two distinctive decay modes: (i) $\bar{b} \to \bar{c}, \bar{u}$ with c quark being a spectator, and (ii) $c \to s, d$ with $\bar{b}$ quark being a spectator. The transition (i) induce the semileptonic B_c decays to charmonium and D mesons, while the transitions (ii) cause the B_c decays to B_s and B mesons. The kinematical ranges of these semileptonic decay modes are substantially different. The square of the four momentum transfer to the lepton pair extends from 0 to $q^2_{\rm max} \approx 10\,\mathrm{GeV}^2$ for the decays to charmonium and $q^2_{\rm max} \approx 18\,\mathrm{GeV}^2$ for decays to D mesons, but only to $q^2_{\rm max} \approx 1\,\mathrm{GeV}^2$ for decays to B and B_s mesons. Thus the kinematical range for the decay mode (i) is appreciably larger than for the decay mode (ii). Otherwise in the B_c rest frame the maximum recoil three momentum of the final charmonium and D meson turns out to be of order of their masses, while that of final B and B_s mesons is much smaller than the meson masses.

The weak B_c decays to charmonium and D mesons were studied at length in our recent paper [3]. Here we consider the weak B_c decays to B_s and B mesons within the relativistic quark model. The model is based on the quasipotential approach in quantum field theory and was fruitfully applied for describing the electroweak decays and mass spectra of heavy-light mesons, heavy quarkonia [4–9] and B_c meson [10]. The relativistic wave functions obtained in the latter paper are used below to calculate the transition matrix elements. The consistent theoretical description of B_c decays requires a reliable determination of the q^2 dependence of the decay amplitudes in the whole kinematical range. In most previous calculations the corresponding decay form factors were determined only at one kinematical point, either $q^2 = 0$ or $q^2 = q^2_{\rm max}$, and then extrapolated to the allowed kinematical range using some phenomenological ansatz (mainly (di)pole or Gaussian). Our aim is to explicitly determine the q^2 dependence of form factors in the whole kinematical range thus avoiding extrapolations and reducing uncertainties.

This paper is organized as follows. In Sect. 2 we describe the underlying relativistic quark model. The method for calculating matrix elements of the weak current for $c \to s, d$ transitions in B_c meson decays is presented in Sect. 3. Special attention is paid to the dependence on the momentum transfer of the decay amplitudes. The B_c decay form factors are calculated in the whole kinematical range in Sect. 4. The q^2 dependence of the form factors is explicitly determined. These form factors are used for the calculation of the B_c semileptonic decay rates in Sect. 5. Section 6 contains our predictions for the energetic non-leptonic B_c decays in the factorization approximation, and a comparison of our results with other theoretical calculations is presented. Our conclusions are given in Sect. 7. Finally, the appendix contains complete expressions for the decay form factors.

2 Relativistic quark model

In the quasipotential approach a meson is described by the wave function of the bound quark–antiquark state, which satisfies the quasipotential equation [11] of the Schrödinger type [12]

$$\left(\frac{b^2(M)}{2\mu_R} - \frac{\mathbf{p}^2}{2\mu_R}\right)\Psi_M(\mathbf{p}) = \int \frac{d^3q}{(2\pi)^3} V(\mathbf{p},\mathbf{q};M)\Psi_M(\mathbf{q}), \tag{1}$$

where the relativistic reduced mass is

$$\mu_R = \frac{E_1E_2}{E_1+E_2} = \frac{M^4-(m_1^2-m_2^2)^2}{4M^3}, \tag{2}$$

and E_1, E_2 are the center of mass energies on mass shell given by

$$E_1 = \frac{M^2-m_2^2+m_1^2}{2M}, \quad E_2 = \frac{M^2-m_1^2+m_2^2}{2M}. \tag{3}$$

Here $M = E_1 + E_2$ is the meson mass, $m_{1,2}$ are the quark masses, and $\mathbf{p}$ is their relative momentum. In the center of mass system the relative momentum squared on mass shell reads

$$b^2(M) = \frac{[M^2-(m_1+m_2)^2][M^2-(m_1-m_2)^2]}{4M^2}. \tag{4}$$

The kernel $V(\mathbf{p},\mathbf{q};M)$ in (1) is the quasipotential operator of the quark–antiquark interaction. It is constructed with the help of the off mass shell scattering amplitude, projected onto the positive-energy states. Constructing the quasipotential of the quark–antiquark interaction, we have assumed that the effective interaction is the sum of the usual one-gluon exchange term with a mixture of long-range vector and scalar linear confining potentials, where the vector confining potential contains the Pauli interaction. The quasipotential is then defined by [4]

$$V(\mathbf{p},\mathbf{q};M) = \bar{u}_1(p)\bar{u}_2(-p)\mathcal{V}(\mathbf{p},\mathbf{q};M)u_1(q)u_2(-q), \tag{5}$$

with

$$\mathcal{V}(\mathbf{p},\mathbf{q};M) = \frac{4}{3}\alpha_s D_{\mu\nu}(\mathbf{k})\gamma_1^\mu\gamma_2^\nu + V^V_{\rm conf}(\mathbf{k})\Gamma_1^\mu\Gamma_{2;\mu} + V^S_{\rm conf}(\mathbf{k}),$$

where α_s is the QCD coupling constant, $D_{\mu\nu}$ is the gluon propagator in the Coulomb gauge

$$D^{00}(\mathbf{k}) = -\frac{4\pi}{\mathbf{k}^2},$$
$$D^{ij}(\mathbf{k}) = -\frac{4\pi}{k^2}\left(\delta^{ij} - \frac{k^ik^j}{\mathbf{k}^2}\right), \quad D^{0i} = D^{i0} = 0, \tag{6}$$

and $\mathbf{k} = \mathbf{p} - \mathbf{q}$; γ_μ and $u(p)$ are the Dirac matrices and spinors

$$u^\lambda(p) = \sqrt{\frac{\epsilon(p)+m}{2\epsilon(p)}}\begin{pmatrix}1\\ \dfrac{\sigma\mathbf{p}}{\epsilon(p)+m}\end{pmatrix}\chi^\lambda. \tag{7}$$

Here σ and χ^λ are the Pauli matrices and spinors; $\epsilon(p) = \sqrt{p^2+m^2}$. The effective long-range vector vertex is given by

$$\Gamma_\mu(\mathbf{k}) = \gamma_\mu + \frac{i\kappa}{2m}\sigma_{\mu\nu}k^\nu, \tag{8}$$

where κ is the Pauli interaction constant characterizing the long-range anomalous chromomagnetic moment of quarks. Vector and scalar confining potentials in the non-relativistic limit reduce to

$$V_V(r) = (1-\varepsilon)Ar + B,$$
$$V_S(r) = \varepsilon Ar, \tag{9}$$

reproducing

$$V_{\rm conf}(r) = V_S(r) + V_V(r) = Ar + B, \tag{10}$$

where ε is the mixing coefficient.

The expression for the quasipotential of the heavy quarkonia, expanded in v^2/c^2 without and with retardation corrections to the confining potential, can be found in [4] and [5, 10], respectively. The structure of the spin-dependent interaction is in agreement with the parameterization of Eichten and Feinberg [13]. The quasipotential for the heavy quark interaction with a light antiquark without employing the expansion in inverse powers of the light quark mass is given in [6]. All the parameters of our model like quark masses, parameters of the linear confining potential A and B, mixing coefficient ε and anomalous chromomagnetic quark moment κ are fixed from the analysis of heavy quarkonium masses [4] and radiative decays [7]. The quark masses $m_b = 4.88$ GeV, $m_c = 1.55$ GeV, $m_s = 0.50$ GeV, $m_{u,d} = 0.33$ GeV and the parameters of the linear potential $A = 0.18\,\mathrm{GeV}^2$ and $B = -0.16$ GeV have the usual values of the quark models. The value of the mixing coefficient of vector and scalar confining potentials $\varepsilon = -1$ has been determined from the consideration of the heavy quark expansion for the semileptonic $B \to D$ decays [8] and charmonium radiative decays [7]. Finally, the universal Pauli interaction constant $\kappa = -1$ has been fixed from the analysis of the fine splitting of heavy quarkonia ${}^3P_J$- states [4]. Note that the long-range magnetic contribution to the potential in our model is proportional to $(1+\kappa)$ and thus vanishes for the chosen value of $\kappa = -1$. It has been known for a long time that the correct reproduction of the spin-dependent part of the quark–antiquark interaction requires either assuming the scalar confinement or equivalently introducing the Pauli interaction with $\kappa = -1$ [4, 5, 14] in the vector confinement.

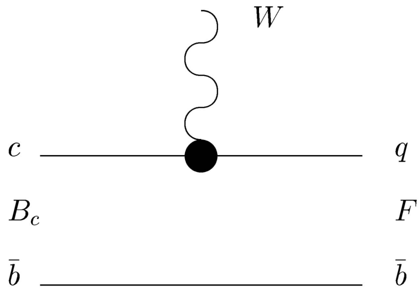

Fig. 1. Lowest order vertex function $\Gamma^{(1)}$ contributing to the current matrix element (11)

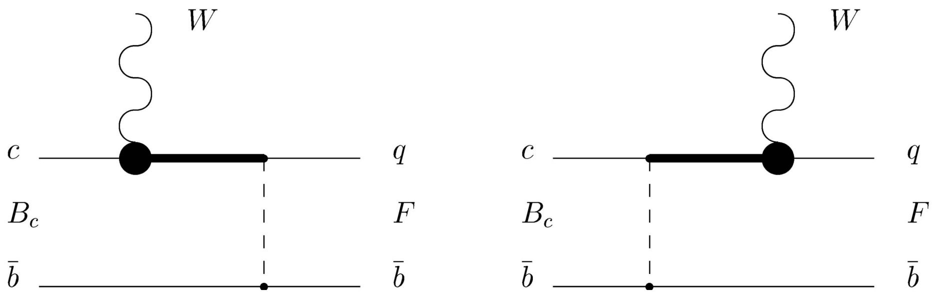

Fig. 2. Vertex function $\Gamma^{(2)}$ taking the quark interaction into account. Dashed lines correspond to the effective potential $\mathcal{V}$ in (5). Bold lines denote the negative-energy part of the quark propagator

3 Matrix elements of the electroweak current for $c \to s, d$ transitions

In order to calculate the exclusive semileptonic decay rate of the B_c meson, it is necessary to determine the corresponding matrix element of the weak current between meson states. In the quasipotential approach, the matrix element of the weak current $J^{\rm W}_\mu = \bar q\gamma_\mu(1-\gamma_5)c$, associated with $c \to q$ ($q = s$ or d) transition, between a B_c meson with mass M_{B_c} and momentum p_{B_c} and a final meson F ($F = B_s^{(*)}$ or $B^{(*)}$) with mass M_F and momentum p_F takes the form [15]

$$\langle F(p_F)|J^{\rm W}_\mu|B_c(p_{B_c})\rangle = \int \frac{{\rm d}^3p\,{\rm d}^3q}{(2\pi)^6}\bar\Psi_{F\,\mathbf{p}_F}(\mathbf{p})\Gamma_\mu(\mathbf{p},\mathbf{q})\Psi_{B_c\,\mathbf{p}_{B_c}}(\mathbf{q}), \qquad (11)$$

where $\Gamma_\mu(\mathbf{p},\mathbf{q})$ is the two-particle vertex function and $\Psi_{M\,\mathbf{p}_M}$ are the meson ($M = B_c, F$) wave functions projected onto the positive-energy states of quarks and boosted to the moving reference frame with momentum $\mathbf{p}_M$.

The contributions to Γ come from Figs. 1 and 2. The contribution $\Gamma^{(2)}$ is the consequence of the projection onto the positive-energy states. Note that the form of the relativistic corrections resulting from the vertex function $\Gamma^{(2)}$ is explicitly dependent on the Lorentz structure of the quark–antiquark interaction. In the leading order of the v^2/c^2 expansion for B_c and in the heavy quark limit $m_b \to \infty$ for B_s, B only $\Gamma^{(1)}$ contributes, while $\Gamma^{(2)}$ contributes already at the subleading order. The vertex functions look like

$$\Gamma^{(1)}_\mu(\mathbf{p},\mathbf{q}) = \bar u_q(p_q)\gamma_\mu(1-\gamma^5)u_c(q_c)(2\pi)^3\delta(\mathbf{p}_b-\mathbf{q}_b), \quad (12)$$

and

$$\Gamma^{(2)}_\mu(\mathbf{p},\mathbf{q}) = \bar u_q(p_q)\bar u_b(p_b) \times\left\{\gamma_{1\mu}(1-\gamma_1^5)\frac{\Lambda_c^{(-)}(k)}{\epsilon_c(k)+\epsilon_c(p_q)}\gamma_1^0\mathcal{V}(\mathbf{p}_b-\mathbf{q}_b) + \mathcal{V}(\mathbf{p}_b-\mathbf{q}_b)\frac{\Lambda_q^{(-)}(k')}{\epsilon_q(k')+\epsilon_q(q_c)}\gamma_1^0\gamma_{1\mu}\left(1-\gamma_1^5\right)\right\} \times u_c(q_c)u_b(q_b), \qquad (13)$$

where the superscripts "(1)" and "(2)" correspond to Figs. 1 and 2, $\mathbf{k} = \mathbf{p}_q - \mathbf{\Delta}$; $\mathbf{k}' = \mathbf{q}_c + \mathbf{\Delta}$; $\mathbf{\Delta} = \mathbf{p}_F - \mathbf{p}_{B_c}$;

$$\Lambda^{(-)}(p) = \frac{\epsilon(p) - (m\gamma^0 + \gamma^0(\boldsymbol{\gamma}\mathbf{p}))}{2\epsilon(p)}.$$

Here [15]

$$p_{q,b} = \epsilon_{q,b}(p)\frac{p_F}{M_F} \pm \sum_{i=1}^3 n^{(i)}(p_F)p^i,$$

$$q_{c,b} = \epsilon_{c,b}(q)\frac{p_{B_c}}{M_{B_c}} \pm \sum_{i=1}^3 n^{(i)}(p_{B_c})q^i,$$

and $n^{(i)}$ are three four-vectors given by

$$n^{(i)\mu}(p) = \left\{\frac{p^i}{M},\ \delta_{ij} + \frac{p^ip^j}{M(E+M)}\right\}, \quad E = \sqrt{\mathbf{p}^2 + M^2}.$$

It is important to note that the wave functions entering the weak current matrix element (11) are not in the rest frame in general. For example, in the B_c meson rest frame ($\mathbf{p}_{B_c} = 0$), the final meson is moving with the recoil momentum $\mathbf{\Delta}$. The wave function of the moving meson $\Psi_{F\,\mathbf{\Delta}}$ is connected with the wave function in the rest frame $\Psi_{F\,\mathbf{0}} \equiv \Psi_F$ by the transformation [15]

$$\Psi_{F\,\mathbf{\Delta}}(\mathbf{p}) = D_q^{1/2}(R^{\rm W}_{L_{\mathbf{\Delta}}})D_b^{1/2}(R^{\rm W}_{L_{\mathbf{\Delta}}})\Psi_{F\,\mathbf{0}}(\mathbf{p}), \qquad (14)$$

where $R^{\rm W}$ is the Wigner rotation, $L_{\mathbf{\Delta}}$ is the Lorentz boost from the meson rest frame to a moving one, and the rotation matrix $D^{1/2}(R)$ in a spinor representation is given by

$$\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} D^{1/2}_{q,b}(R^{\rm W}_{L_{\mathbf{\Delta}}}) = S^{-1}(\mathbf{p}_{q,b})S(\mathbf{\Delta})S(\mathbf{p}), \qquad (15)$$

where

$$S(\mathbf{p}) = \sqrt{\frac{\epsilon(p)+m}{2m}}\left(1 + \frac{\boldsymbol{\alpha}\mathbf{p}}{\epsilon(p)+m}\right)$$

is the usual Lorentz transformation matrix of the four-spinor.

The general structure of the current matrix element (11) is rather complicated, because it is necessary to integrate both with respect to ${\rm d}^3p$ and ${\rm d}^3q$. The δ-function in the expression (12) for the vertex function $\Gamma^{(1)}$ permits one to perform one of these integrations. As a result the contribution of $\Gamma^{(1)}$ to the current matrix element has the usual structure of an overlap integral of meson wave functions and can be calculated exactly (without employing any expansion) in the whole kinematical range, if the wave functions of the initial and final mesons are known. The situation with the contribution $\Gamma^{(2)}$ is different. Here, instead of a δ-function, we have a complicated structure, containing the potential of the $q\bar{q}$ interaction in a meson. Thus in the general case we cannot accomplish one of the integrations in the contribution of $\Gamma^{(2)}$ to the matrix element (11). Therefore, one should use some additional considerations in order to simplify calculations. The main idea is to expand the vertex function $\Gamma^{(2)}$, given by (13), in such a way that it will be possible to use the quasipotential equation (1) in order to perform one of the integrations in the current matrix element (11).

The natural expansion parameters for B_c decays to B_s, B mesons are the active c and spectator b quark masses. However, the heavy c quark undergoes a weak transition to the light s or d quark. The constituent s, d quark masses are of the same order of magnitude as the relative momentum and binding energy, thus we cannot apply the expansion in inverse powers of their masses. The heavy quark expansion in $1/m_{c,b}$ significantly simplifies the structure of the $\Gamma^{(2)}$ contribution to the decay matrix element, but the momentum $\mathbf{p}$ dependence of the light quark energies $\epsilon_q(p)$ still prevents to perform one of the integrations. It is important to note that the kinematically allowed range for B_c decays to B_s and B meson is not large ($|\boldsymbol{\Delta}_{\max}| = (M_{B_c}^2 - M_F^2)/(2M_{B_c}) \sim 0.8\,\mathrm{GeV}$ for decays to B_s and $\sim 0.9\,\mathrm{GeV}$ for decays to B mesons). This means that the recoil momentum $\boldsymbol{\Delta}$ of a final meson is of the same order as the relative momentum $\mathbf{p}$ of quarks inside a heavy-light meson ($\sim 0.5\,\mathrm{GeV}$) in the whole kinematical range. Taking also into account that the final B_s and B mesons are weakly bound [6], [1] we can replace the light quark energies by the center of mass energies on mass shell $\epsilon_q(p) \to E_q = (M_F^2 - m_b^2 + m_q^2)/(2M_F)$. We used such a substitution in our analysis of heavy-light meson mass spectra [6] which allowed us to treat the light quark relativistically without an unjustified expansion in inverse powers of its mass. Making these replacements and expansions we see that it is possible to integrate the current matrix element (11) either with respect to d^3p or d^3q using the quasipotential equation (1). Performing integrations and taking the sum of the contributions $\Gamma^{(1)}$ and $\Gamma^{(2)}$ we get the expression for the current matrix element, which contains ordinary overlap integrals of meson wave functions and is valid in the whole kinematical range. Hence the matrix element can be easily calculated using numerical wave functions found in our analysis of the meson mass spectra [5, 10].

[1] The sum of constituent quark masses $m_b + m_q$ is very close to the ground state meson mass M.

4 $\mathrm{B_c}$ decay form factors

The matrix elements of the weak current J^{W} for B_c decays to pseudoscalar mesons ($P = B_s, B$) can be parametrized by two invariant form factors:

$$\begin{aligned}
&\langle P(p_F)|\bar{q}\gamma^\mu c|B_c(p_{B_c})\rangle \\
&= f_+(q^2)\left[p_{B_c}^\mu + p_F^\mu - \frac{M_{B_c}^2 - M_P^2}{q^2}\, q^\mu\right] \\
&+ f_0(q^2)\frac{M_{B_c}^2 - M_P^2}{q^2}\, q^\mu, \qquad (16)
\end{aligned}$$

where $q = p_{B_c} - p_F$; M_{B_c} is the B_c meson mass and M_P is the pseudoscalar meson mass.

The corresponding matrix elements for B_c decays to vector mesons ($V = B_s^*, B^*$) are parametrized by four form factors

$$\langle V(p_F)|\bar{q}\gamma^\mu c|B(p_{B_c})\rangle = \frac{2\mathrm{i}V(q^2)}{M_{B_c} + M_V}\epsilon^{\mu\nu\rho\sigma}\epsilon^*_\nu p_{B_c\rho} p_{F\sigma} \qquad (17)$$

$$\begin{aligned}
&\langle V(p_F)|\bar{q}\gamma^\mu\gamma_5 c|B(p_{B_c})\rangle \\
&= 2M_V A_0(q^2)\frac{\epsilon^*\cdot q}{q^2}\, q^\mu \\
&\quad + (M_{B_c} + M_V)A_1(q^2)\left(\epsilon^{*\mu} - \frac{\epsilon^*\cdot q}{q^2}\, q^\mu\right) \qquad (18) \\
&\quad - A_2(q^2)\frac{\epsilon^*\cdot q}{M_{B_c} + M_V}\left[p_{B_c}^\mu + p_F^\mu - \frac{M_{B_c}^2 - M_V^2}{q^2}\, q^\mu\right],
\end{aligned}$$

where M_V and ϵ_μ are the mass and polarization vector of the final vector meson. The following relations hold for the form factors at the maximum recoil point of the final meson ($q^2 = 0$):

$$f_+(0) = f_0(0),$$

$$A_0(0) = \frac{M_{B_c} + M_V}{2M_V}A_1(0) - \frac{M_{B_c} - M_V}{2M_V}A_2(0).$$

In the limit of vanishing lepton mass, the form factors f_0 and A_0 do not contribute to the semileptonic decay rates. However, they contribute to the non-leptonic decay rates in the factorization approximation.

It is convenient to consider B_c semileptonic and nonleptonic decays in the B_c meson rest frame. Then it is important to take into account the boost of the final meson wave function from the rest reference frame to the moving one with the recoil momentum $\boldsymbol{\Delta}$, given by (14). Now we can apply the method for calculating decay matrix elements described in the previous section. As it is argued above, the leading contributions arising from the vertex function $\Gamma^{(1)}$ can be exactly expressed through the overlap integrals of the meson wave functions in the whole kinematical range. For the subleading contribution $\Gamma^{(2)}$, the expansion in powers of the ratio of the relative quark momentum $\mathbf{p}$ to heavy quark masses $m_{b,c}$ should be performed. Taking into account that the recoil momentum of

Table 1. Form factors of weak B_c decays ($c \to s(d)$ transitions)

Transition	$f_+(q^2)$	$f_0(q^2)$	$V(q^2)$	$A_1(q^2)$	$A_2(q^2)$	$A_0(q^2)$
$B_c \to B_s(B_s^*)$						
$q^2 = q^2_{\rm max}$	0.99	0.86	6.25	0.76	2.62	0.91
$q^2 = 0$	0.50	0.50	3.44	0.49	2.19	0.35
$B_c \to B(B^*)$						
$q^2 = q^2_{\rm max}$	0.96	0.80	8.91	0.72	2.83	1.06
$q^2 = 0$	0.39	0.39	3.94	0.42	2.89	0.20

the final meson $\mathbf{\Delta}$ is not large we replace the final light quark energies $\epsilon_q(p)$ by the center of mass energies on mass shell E_q. Such a replacement is well justified near the point of zero recoil of the final B_s, B meson. The weak dependence of this subleading contribution on the recoil momentum and its numerical smallness due to its proportionality to the small meson binding energy permits its extrapolation to the whole kinematical range. As a result, we get the following expressions for the B_c decay form factors:

(a) $B_c \to P$ transitions ($P = B_s, B$)

$$f_+(q^2) = f_+^{(1)}(q^2) + \varepsilon f_+^{S(2)}(q^2) + (1-\varepsilon) f_+^{V(2)}(q^2), \quad (19)$$

$$f_0(q^2) = f_0^{(1)}(q^2) + \varepsilon f_0^{S(2)}(q^2) + (1-\varepsilon) f_0^{V(2)}(q^2), \quad (20)$$

(b) $B_c \to V$ transitions ($V = B_s^*, B^*$)

$$V(q^2) = V^{(1)}(q^2) + \varepsilon V^{S(2)}(q^2) + (1-\varepsilon) V^{V(2)}(q^2), \quad (21)$$

$$A_1(q^2) = A_1^{(1)}(q^2) + \varepsilon A_1^{S(2)}(q^2) + (1-\varepsilon) A_1^{V(2)}(q^2), \quad (22)$$

$$A_2(q^2) = A_2^{(1)}(q^2) + \varepsilon A_2^{S(2)}(q^2) + (1-\varepsilon) A_2^{V(2)}(q^2), \quad (23)$$

$$A_0(q^2) = A_0^{(1)}(q^2) + \varepsilon A_0^{S(2)}(q^2) + (1-\varepsilon) A_0^{V(2)}(q^2), \quad (24)$$

where $f_{+,0}^{(1)}$, $f_{+,0}^{S,V(2)}$, $A_{0,1,2}^{(1)}$, $A_{0,1,2}^{S,V(2)}$, $V^{(1)}$ and $V^{S,V(2)}$ are given in the appendix. The superscripts "(1)" and "(2)" correspond to Figs. 1 and 2, S and V to the scalar and vector potentials of the $q\bar{q}$ interaction. The mixing parameter of scalar and vector confining potentials ε is fixed to be -1 in our model.

It is easy to check that in the heavy quark limit the decay matrix elements (16)–(18) with form factors (19)–(24) satisfy the heavy quark spin symmetry relations [16] obtained near the zero recoil point ($\mathbf{\Delta} \to 0$).

For numerical calculations we use the quasipotential wave functions of the B_c meson, and the B_s and B mesons, obtained in the mass spectrum calculations [5, 6]. Our model predicts the B_c meson mass $M_{B_c} = 6.270$ GeV [10], while for the $B_s^{(*)}$ and $B^{(*)}$ meson masses we use the experimental data [17]. The calculated values of the form factors at zero ($q^2 = q^2_{\rm max}$) and maximum ($q^2 = 0$) recoil of the final meson are listed in Table 1. In Fig. 3 we plot leading $f_+^{(1)}$ and subleading $f_+^{S(2)}$, $f_+^{V(2)}$ contributions to the form factor f_+ for the $B_c \to B_s$ transition, as an example. We see that the leading contribution $f_+^{(1)}$ is dominant in the whole kinematical range, as it was expected. The subleading contributions $f_+^{S(2)}$, $f_+^{V(2)}$ are small and weakly depend on q^2. The behavior of the corresponding contributions to the other form factors is similar. This supports our conjecture that the formulae (38)–(55) can be applied for the calculation of the form factors of $B_c \to B_s(B)^{(*)}$ transitions in the whole kinematical range.

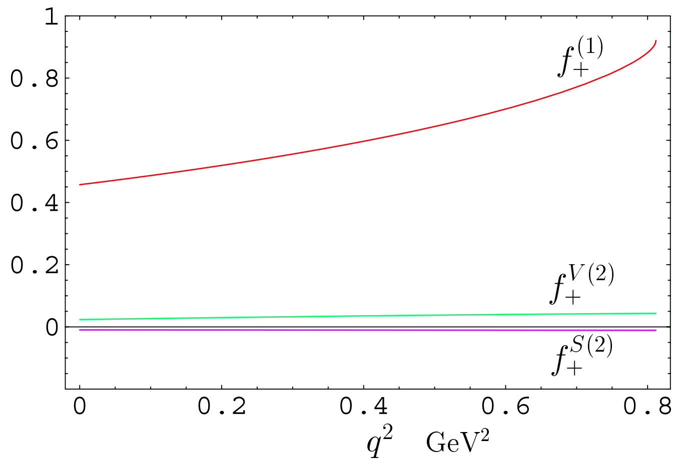

Fig. 3. Leading $f_+^{(1)}$ and subleading $f_+^{S(2)}$, $f_+^{V(2)}$ contributions to the form factor f_+ for the $B_c \to B_s$ transition

In Figs. 4–7 we plot the calculated q^2 dependence of the weak form factors of the Cabibbo–Kobayashi–Maskawa (CKM) favored ($B_c \to B_s$, $B_c \to B_s^*$), as well as CKM suppressed ($B_c \to B$, $B_c \to B^*$) transitions in the whole kinematical range.

In the following sections we use the obtained form factors for the calculation of the semileptonic and non-leptonic B_c decay rates.

5 Semileptonic decays

The differential semileptonic decay rates can be expressed in terms of the form factors as follows.

(a) $B_c \to Pe\nu$ decays ($P = B_s, B$)

$$\frac{{\rm d}\Gamma}{{\rm d}q^2}(B_c \to Pe\nu) = \frac{G_{\rm F}^2 \Delta^3 |V_{cq}|^2}{24\pi^3} |f_+(q^2)|^2. \quad (25)$$

(b) $B_c \to Ve\nu$ decays ($V = B_s^*, B^*$)

$$\frac{{\rm d}\Gamma}{{\rm d}q^2}(B_c \to Ve\nu) \quad (26)$$
$$= \frac{G_{\rm F}^2 \Delta |V_{cq}|^2}{96\pi^3} \frac{q^2}{M_{B_c}^2} \left(|H_+(q^2)|^2 + |H_-(q^2)|^2 + |H_0(q^2)|^2\right),$$

where $G_{\rm F}$ is the Fermi constant, V_{cq} is the CKM matrix element ($q = s, d$),

$$\Delta \equiv |\mathbf{\Delta}| = \sqrt{\frac{(M_{B_c}^2 + M_{P,V}^2 - q^2)^2}{4M_{B_c}^2} - M_{P,V}^2}.$$

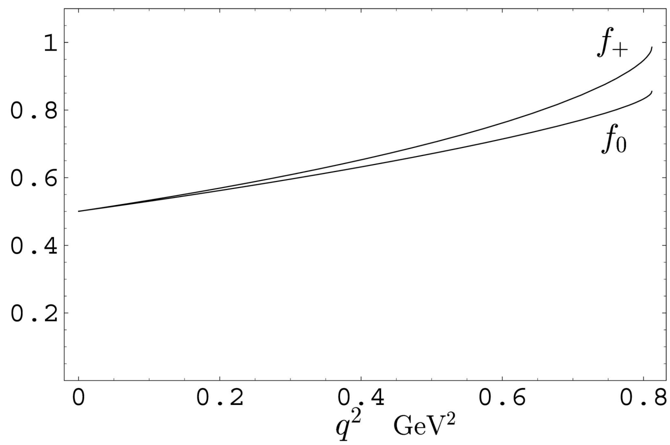

Fig. 4. Form factors of the $B_c \to B_s e\nu$ decay

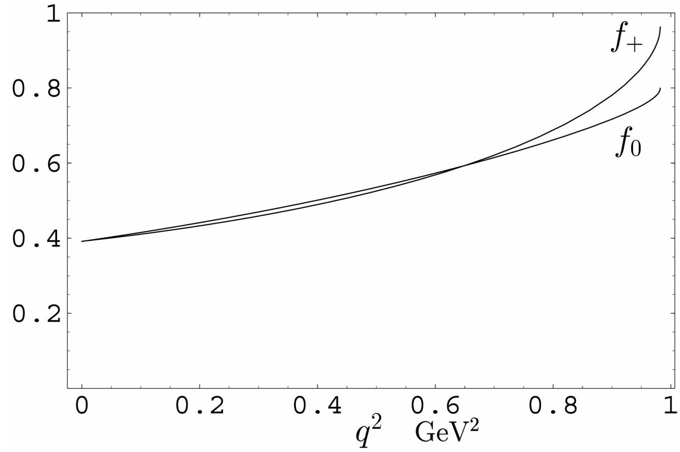

Fig. 6. Form factors of the $B_c \to B e\nu$ decay

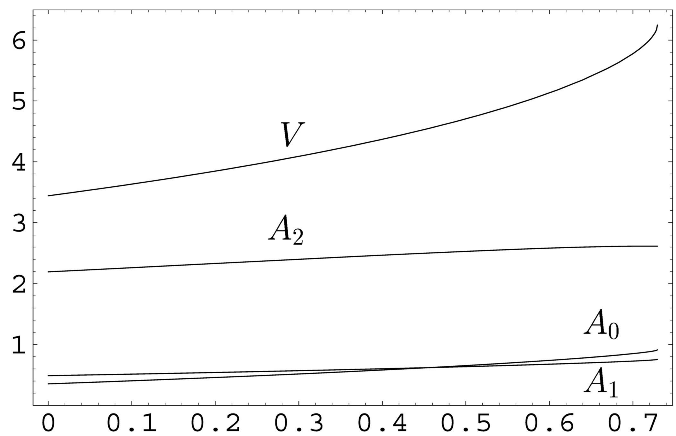

Fig. 5. Form factors of the $B_c \to B_s^* e\nu$ decay

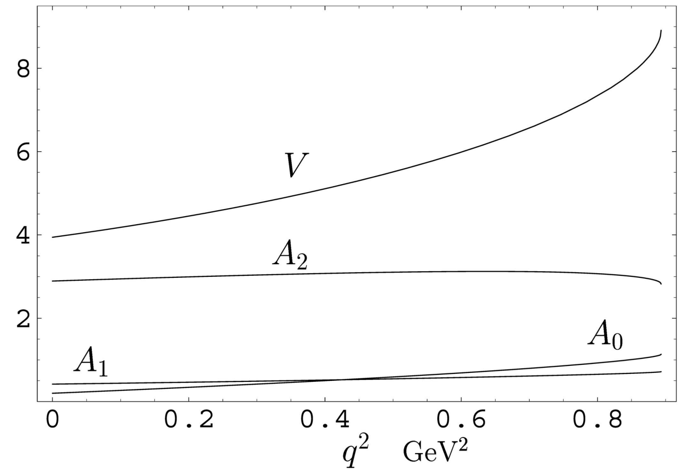

Fig. 7. Form factors of the $B_c \to B^* e\nu$ decay

The helicity amplitudes are given by

$$H_\pm(q^2) = \frac{2M_{B_c}\Delta}{M_{B_c}+M_V}\left[V(q^2) \mp \frac{(M_{B_c}+M_V)^2}{2M_{B_c}\Delta}A_1(q^2)\right], \tag{27}$$

$$H_0(q^2) = \frac{1}{2M_V\sqrt{q^2}} \times \left[(M_{B_c}+M_V)(M_{B_c}^2 - M_V^2 - q^2)A_1(q^2) - \frac{4M_{B_c}^2\Delta^2}{M_{B_c}+M_V}A_2(q^2)\right]. \tag{28}$$

The decay rates to the longitudinally and transversely polarized vector mesons are defined by

$$\frac{\mathrm{d}\Gamma_L}{\mathrm{d}q^2} = \frac{G_\mathrm{F}^2\Delta|V_{cq}|^2}{96\pi^3}\frac{q^2}{M_{B_c}^2}|H_0(q^2)|^2, \tag{29}$$

$$\frac{\mathrm{d}\Gamma_T}{\mathrm{d}q^2} = \frac{\mathrm{d}\Gamma_+}{\mathrm{d}q^2} + \frac{\mathrm{d}\Gamma_-}{\mathrm{d}q^2} \tag{30}$$

$$= \frac{G_\mathrm{F}^2\Delta|V_{cq}|^2}{96\pi^3}\frac{q^2}{M_{B_c}^2}\left(|H_+(q^2)|^2 + |H_-(q^2)|^2\right).$$

In Figs. 8–11 we plot the differential semileptonic decay rates $\mathrm{d}\Gamma/\mathrm{d}q^2$ for semileptonic decays $B_c \to B_s^{(*)}e\nu$ and $B_c \to B^{(*)}e\nu$ calculated in our model using (25) and (26) both with and without account of $1/m_{b,c}$ corrections to the decay form factors (38)–(55). [2] From these plots we see that relativistic effects related to heavy quarks increase the rates of semileptonic B_c decays to the pseudoscalar B_s and B mesons and decrease the rates of semileptonic decays to vector B_s^* and B^* mesons.

We calculate the total rates of the semileptonic B_c decays to the $B_s^{(*)}$ and $B^{(*)}$ mesons integrating the corresponding differential decay rates over q^2. For calculations we use the following values of the CKM matrix elements: $|V_{cs}| = 0.974$, $|V_{cd}| = 0.223$. The results are given in Table 2 in comparison with predictions of other approaches based on quark models [18, 20, 21, 23, 24, 26], QCD sum rules [19] and on the application of heavy quark symmetry relations [22, 25] to the quark model. Our predic-

[2] Relativistic wave functions were used for both calculations.

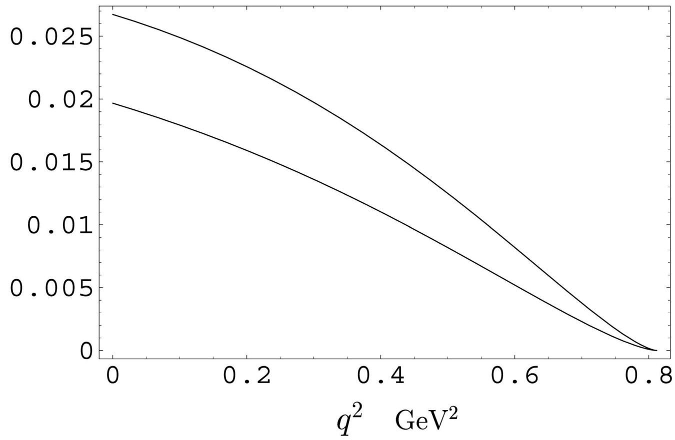

Fig. 8. Differential decay rates $(1/|V_{cs}|^2)\mathrm{d}\Gamma/\mathrm{d}q^2$ of $B_c \to B_s e\nu$ decay (in $10^{-12}\,\mathrm{GeV}^{-1}$). The lower curve is evaluated without account of $1/m_{b,c}$ corrections

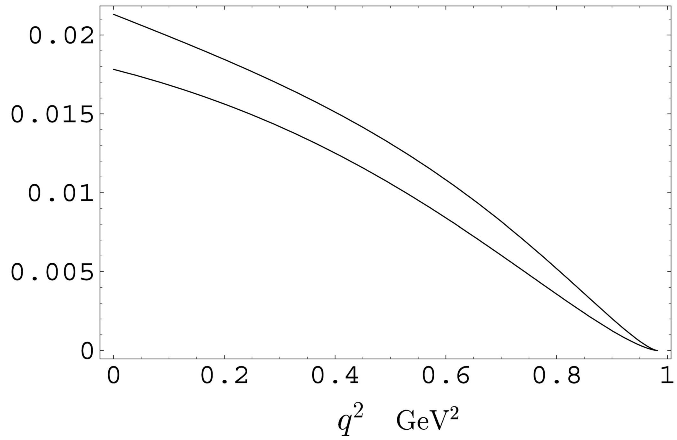

Fig. 10. Differential decay rate $(1/|V_{cd}|^2)\mathrm{d}\Gamma/\mathrm{d}q^2$ of $B_c \to Be\nu$ decay (in $10^{-12}\,\mathrm{GeV}^{-1}$). The lower curve is evaluated without account of $1/m_{b,c}$ corrections

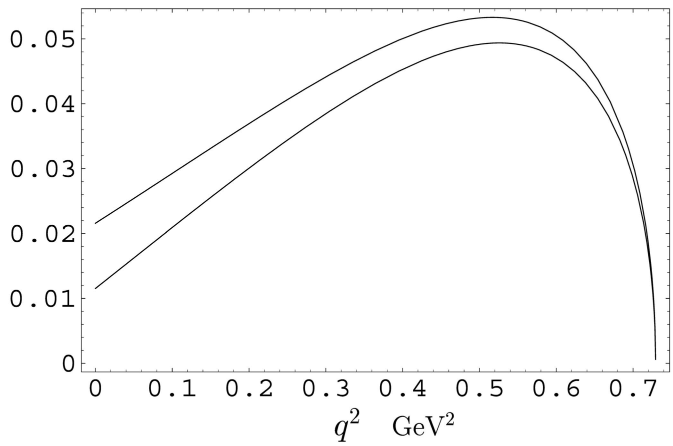

Fig. 9. Differential decay rates $(1/|V_{cs}|^2)\mathrm{d}\Gamma/\mathrm{d}q^2$ of $B_c \to B_s^* e\nu$ decay (in $10^{-12}\,\mathrm{GeV}^{-1}$). The upper curve is evaluated without account of $1/m_{b,c}$ corrections

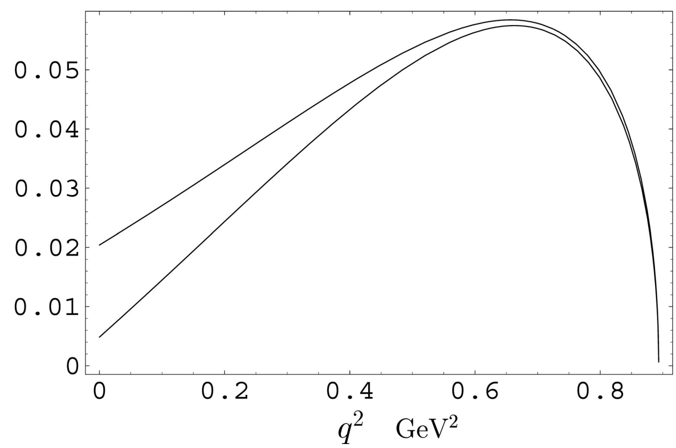

Fig. 11. Differential decay rates $(1/|V_{cd}|^2)\mathrm{d}\Gamma/\mathrm{d}q^2$ of $B_c \to B^* e\nu$ decay (in $10^{-12}\,\mathrm{GeV}^{-1}$). The upper curve is evaluated without account of $1/m_{b,c}$ corrections

Table 2. Semileptonic decay rates Γ of B_c to $B_s^{(*)}$ and $B^{(*)}$ mesons (in $10^{-15}\,\mathrm{GeV}$)

Decay	Our result	[18]	[19]	[20]	[21]	[22]	[23]	[24]	[25]	[26]
$B_c \to B_s e\nu$	12	29	59	14.3	26.6	11.1(12.9)	15	12.3	11.75	26.8
$B_c \to B_s^* e\nu$	25	37	65	50.4	44.0	33.5(37.0)	34	19.0	32.56	34.6
$B_c \to Be\nu$	0.6	2.1	4.9	1.14	2.30	0.9(1.0)			0.59	1.90
$B_c \to B^* e\nu$	1.7	2.3	8.5	3.53	3.32	2.8(3.2)			2.44	2.34

tions for the CKM favored semileptonic B_c decays to $B_s^{(*)}$ are smaller than those of QCD sum rules [19] and quark models [18, 20, 21, 26], but agree with quark model results [22–25]. For the CKM suppressed semileptonic decays of B_c to $B^{(*)}$ mesons our results are in agreement with the ones based on the application of heavy quark symmetry relations [22, 25] to the quark model.

In Table 3 we present for completeness our predictions for the rates of the semileptonic B_c decays to vector (B_s^* and B^*) mesons with longitudinal (L) or transverse (T) polarization and to the states with helicities $\lambda = \pm 1$, as well as their ratios.

Table 3. Semileptonic decay rates $\Gamma_{\mathrm{L,T,+,-}}$ (in $10^{-15}\,\mathrm{GeV}$) and their ratios for B_c decays to vector B_s^* and B^* mesons

Decay	Γ_L	Γ_T	$\Gamma_\mathrm{L}/\Gamma_\mathrm{T}$	Γ_+	Γ_-	Γ_+/Γ_-
$B_c \to B_s^* e\nu$	10.5	14.5	0.74	3.1	11.4	0.27
$B_c \to B^* e\nu$	0.57	1.13	0.50	0.13	1.0	0.13

6 Non-leptonic decays

In the standard model non-leptonic B_c decays are described by the effective Hamiltonian, obtained by integrating out the heavy W-boson and top quark. For the case of

$c \to s, d$ transitions, one gets

$$H_{\rm eff} = \frac{G_{\rm F}}{\sqrt{2}} V_{cs} \left[c_1(\mu) O_1^{cs} + c_2(\mu) O_2^{cs}\right] + \frac{G_{\rm F}}{\sqrt{2}} V_{cd} \left[c_1(\mu) O_1^{cd} + c_2(\mu) O_2^{cd}\right] + \dots \quad (31)$$

The Wilson coefficients $c_{1,2}(\mu)$ are evaluated perturbatively at the W scale and then are evolved down to the renormalization scale $\mu \approx m_c$ by the renormalization-group equations. The ellipsis denote the penguin operators, the Wilson coefficients of which are numerically much smaller than $c_{1,2}$. The local four-quark operators O_1 and O_2 are given by

$$O_1^{cq} = (\tilde{d}u)_{V-A}(\bar{c}q)_{V-A}, \quad (32)$$
$$O_2^{cq} = (\bar{c}u)_{V-A}(\tilde{d}q)_{V-A}, \qquad q = (s, d),$$

where the rotated antiquark field is

$$\tilde{d} = V_{ud}\bar{d} + V_{us}\bar{s}, \quad (33)$$

and for the hadronic current the following notation is used:

$$(\bar{q}q')_{V-A} = \bar{q}\gamma_\mu(1-\gamma_5)q' \equiv J_\mu^{\rm W}.$$

The factorization approach, which is extensively used for the calculation of two-body non-leptonic decays, such as $B_c \to FM$, assumes that the non-leptonic decay amplitude reduces to the product of a form factor and a decay constant [27]. This assumption in general cannot be exact. However, it is expected that factorization can hold for the energetic decays, where the final F meson is heavy and the M meson is light [28]. A justification of this assumption is usually based on the issue of color transparency [29]. In these decays the final hadrons are produced in the form of point-like color-singlet objects with a large relative momentum. And thus the hadronization of the decay products occurs after they are too far away for strongly interacting with each other. That provides the possibility to avoid the final state interaction. A more general treatment of factorization is given in [30, 31].

In this paper we consider the following two types of non-leptonic decays:

(a) $B_c^+ \to B_s^{(*)}(B^{(*)0})M^+$ and
(b) $B_c^+ \to B^{(*)+}M^0$, where the final light M^+ and M^0 mesons are π, ρ or $K^{(*)}$. The corresponding diagrams are shown in Fig. 12, where $q, q_1 = d$, s and $q_2 = u$. Then in the factorization approximation the decay amplitudes can be expressed through the product of one-particle matrix elements

$$\langle F^0 M^+ | H_{\rm eff} | B_c^+ \rangle \quad (34)$$
$$= \frac{G_{\rm F}}{\sqrt{2}} V_{cq} V_{q_1q_2} a_1 \langle F | (\bar{c}q)_{V-A} | B_c \rangle \langle M | (\bar{q}_1 q_2)_{V-A} | 0 \rangle,$$
$$\langle B^{(*)+} M^0 | H_{\rm eff} | B_c^+ \rangle \quad (35)$$
$$= \frac{G_{\rm F}}{\sqrt{2}} V_{cq} V_{q_1q_2} a_2 \langle B^{(*)} | (\bar{c}q_2)_{V-A} | B_c \rangle \langle M | (\bar{q}_1 q)_{V-A} | 0 \rangle,$$

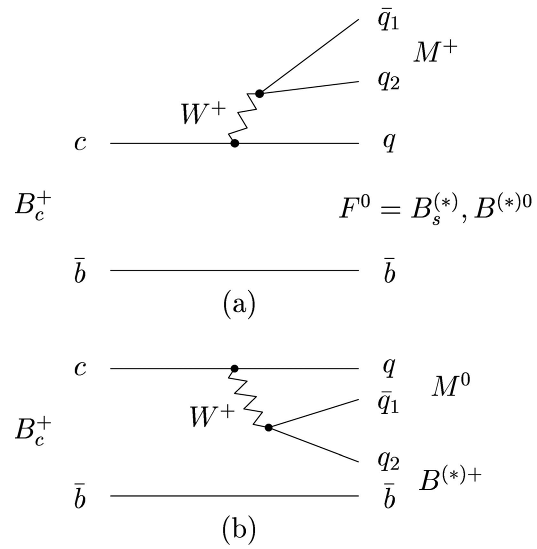

Fig. 12. Quark diagrams for the non-leptonic B_c decays: **a** $B_c^+ \to F^0M^+$ decay; **b** $B_c^+ \to B^+M^0$ decay

where

$$a_1 = c_1(\mu) + \frac{1}{N_c}c_2(\mu), \qquad a_2 = c_2(\mu) + \frac{1}{N_c}c_1(\mu), \quad (36)$$

and N_c is the number of colors.

The matrix element of the current $J_\mu^{\rm W}$ between vacuum and final pseudoscalar (P) or vector (V) meson is parametrized by the decay constants $f_{P,V}$

$$\langle P | \bar{q}_1 \gamma^\mu \gamma_5 q_2 | 0 \rangle = {\rm i} f_P p_P^\mu, \quad \langle V | \bar{q}_1 \gamma_\mu q_2 | 0 \rangle = \epsilon_\mu M_V f_V. \quad (37)$$

We use the following values of the decay constants: $f_\pi = 0.131\,\text{GeV}$, $f_\rho = 0.208\,\text{GeV}$, $f_K = 0.160\,\text{GeV}$ and $f_{K^*} = 0.214\,\text{GeV}$. The CKM matrix elements are $|V_{ud}| = 0.975$, $|V_{us}| = 0.222$.

The matrix elements of the weak current between the B_c meson and the final $B_s^{(*)}$, B meson entering the factorized non-leptonic decay amplitude (34) are parametrized by the set of decay form factors defined in (16) and (17). Using the form factor values obtained in Sect. 4, we get predictions for the non-leptonic $B_c^+ \to F^0M^+$ and $B_c^+ \to B^+M^0$ decay rates and give them in Table 4 in comparison with other calculations [19–23, 26].

In Tables 2 and 4 we confront the predictions of our model for semileptonic and non-leptonic B_c decays with previous calculations [18–26]. The constituent quark models of [18, 24] are based on the same effective quark–meson Lagrangian but use different phenomenological parameterizations (Gaussian [18] and dipole [24]) for the vertex functions, which are assumed to depend only on the loop momentum flowing through the vertex. The relativistic quark models of [20, 21, 26] use different reductions of the Bethe–Salpeter (BS) equation. The authors of [21] apply a non-relativistic instantaneous approximation to the decay matrix elements and relate the BS wave functions to

Table 4. Non-leptonic decay rates Γ (in 10^{-15} GeV)

Decay	Our result	[19]	[20]	[21]	[23]	[22]	[26]
$B_c^+ \to B_s\pi^+$	$25a_1^2$	$167a_1^2$	$15.8a_1^2$	$58.4a_1^2$	$34.8a_1^2$	$30.6a_1^2$	$65.1a_1^2$
$B_c^+ \to B_s\rho^+$	$14a_1^2$	$72.5a_1^2$	$39.2a_1^2$	$44.8a_1^2$	$23.6a_1^2$	$13.6a_1^2$	$42.7a_1^2$
$B_c^+ \to B_s^*\pi^+$	$16a_1^2$	$66.3a_1^2$	$12.5a_1^2$	$51.6a_1^2$	$19.8a_1^2$	$35.6a_1^2$	$25.3a_1^2$
$B_c^+ \to B_s^*\rho^+$	$110a_1^2$	$204a_1^2$	$171a_1^2$	$150a_1^2$	$123a_1^2$	$110.1a_1^2$	$139.6a_1^2$
$B_c^+ \to B_sK^+$	$2.1a_1^2$	$10.7a_1^2$	$1.70a_1^2$	$4.20a_1^2$		$2.15a_1^2$	$4.69a_1^2$
$B_c^+ \to B_sK^{*+}$	$0.03a_1^2$		$1.06a_1^2$			$0.043a_1^2$	$0.296a_1^2$
$B_c^+ \to B_s^*K^+$	$1.1a_1^2$	$3.8a_1^2$	$1.34a_1^2$	$2.96a_1^2$		$1.6a_1^2$	$1.34a_1^2$
$B_c^+ \to B^0\pi^+$	$1.0a_1^2$	$10.6a_1^2$	$1.03a_1^2$	$3.30a_1^2$	$1.50a_1^2$	$1.97a_1^2$	$3.64a_1^2$
$B_c^+ \to B^0\rho^+$	$1.3a_1^2$	$9.7a_1^2$	$2.81a_1^2$	$5.97a_1^2$	$1.93a_1^2$	$1.54a_1^2$	$4.03a_1^2$
$B_c^+ \to B^{*0}\pi^+$	$0.26a_1^2$	$9.5a_1^2$	$0.77a_1^2$	$2.90a_1^2$	$0.78a_1^2$	$2.4a_1^2$	$1.22a_1^2$
$B_c^+ \to B^{*0}\rho^+$	$6.8a_1^2$	$26.1a_1^2$	$9.01a_1^2$	$11.9a_1^2$	$6.78a_1^2$	$8.6a_1^2$	$8.16a_1^2$
$B_c^+ \to B^0K^+$	$0.09a_1^2$	$0.70a_1^2$	$0.105a_1^2$	$0.255a_1^2$		$0.14a_1^2$	$0.272a_1^2$
$B_c^+ \to B^0K^{*+}$	$0.04a_1^2$	$0.15a_1^2$	$0.125a_1^2$	$0.180a_1^2$		$0.032a_1^2$	$0.0965a_1^2$
$B_c^+ \to B^{*0}K^+$	$0.04a_1^2$	$0.56a_1^2$	$0.064a_1^2$	$0.195a_1^2$		$0.12a_1^2$	$0.0742a_1^2$
$B_c^+ \to B^{*0}K^{*+}$	$0.33a_1^2$	$0.59a_1^2$	$0.665a_1^2$	$0.374a_1^2$		$0.34a_1^2$	$0.378a_1^2$
$B_c^+ \to B^+\bar{K}^0$	$34a_2^2$	$286a_2^2$	$39.1a_2^2$	$96.5a_2^2$	$24.0a_2^2$		$103.4a_2^2$
$B_c^+ \to B^+\bar{K}^{*0}$	$13a_2^2$	$64a_2^2$	$46.8a_2^2$	$68.2a_2^2$	$13.8a_2^2$		$36.6a_2^2$
$B_c^+ \to B^{*+}\bar{K}^0$	$15a_2^2$	$231a_2^2$	$24.0a_2^2$	$73.3a_2^2$	$8.9a_2^2$		$28.9a_2^2$
$B_c^+ \to B^{*+}\bar{K}^{*0}$	$120a_2^2$	$242a_2^2$	$247a_2^2$	$141a_2^2$	$82.3a_2^2$		$143.6a_2^2$
$B_c^+ \to B^+\pi^0$	$0.5a_2^2$	$5.3a_2^2$	$0.51a_2^2$	$1.65a_2^2$	$1.03a_2^2$		
$B_c^+ \to B^+\rho^0$	$0.7a_2^2$	$4.4a_2^2$	$1.40a_2^2$	$2.98a_2^2$	$1.28a_2^2$		
$B_c^+ \to B^{*+}\pi^0$	$0.13a_2^2$	$4.8a_2^2$	$0.38a_2^2$	$1.45a_2^2$	$0.53a_2^2$		
$B_c^+ \to B^{*+}\rho^0$	$3.4a_2^2$	$13.1a_2^2$	$4.50a_2^2$	$5.96a_2^2$	$4.56a_2^2$		

the Schrödinger ones. They do not give sufficient information about the quark interaction potential in their model. The quark models [20, 26] are based on the instantaneous approximation and different versions of the quasipotential equation. The meson wave functions are obtained by solving these equations with the one-gluon exchange plus the long-range scalar linear potentials. The light-front relativistic quark model is used in [23]. The decay form factors are expressed through the overlap integrals of the meson light-front wave functions which are related to the equal-time wave functions. The latter are expressed through the Gaussian functions. The heavy quark spin symmetry relations [16] and constituent quark models are used in [22, 25]. This symmetry permits one to relate the B_c weak decay form factors to a few invariant functions near the zero recoil point. These invariant functions are determined from the wave equation with the Richardson potential [22] or by the Gaussian wave functions [25]. Then they are extrapolated to the whole kinematical range accessible in B_c decays. The authors of [19] employ three-point QCD sum rules taking into account the Coulomb-like α_s/v corrections. The values of the form factors are determined in the vicinity of $q^2 = 0$ and then are extrapolated to the allowed kinematical region using the pole ansatz.

Our relativistic quark model provides the selfconsistent dynamical approach for the calculation of various meson properties. The meson wave functions in this approach are obtained as the solutions of the relativistic quasipotential equation. The weak decay matrix elements are expressed in terms of these wave functions. It allows us to determine explicitly the q^2 dependence of the form factors of the weak B_c decays in the whole kinematical range. All relevant relativistic effects are taken into account (including the boost of the wave functions to the moving reference frame). The light quarks in the final heavy-light meson are treated relativistically. All that increases the reliability of the obtained results.

As one sees from Tables 2 and 4 the theoretical predictions for B_c weak decay rates differ substantially. Thus experimental measurements of corresponding decay rates can discriminate between various approaches.

7 Conclusions

In this paper we considered weak semileptonic and non-leptonic B_c decays to B_s and B mesons, associated with the $c \to s, d$ quark transition, in the framework of the relativistic quark model based on the quasipotential approach in quantum field theory. The weak decay form factors were calculated explicitly in the whole kinematical range using the heavy quark expansion for the initial active quark c and spectator quark $\bar{b}$. The final quark s or d was treated completely relativistically without applying an unjustified expansion in inverse powers of its mass. The leading order contribution of the heavy quark expansion was treated exactly, while in calculating the subleading order contribution the replacement of light quark energies

Table 5. Branching fractions (in %) of exclusive B_c decays calculated for the fixed values of the B_c lifetime $\tau_{B_c} = 0.46$ ps and $a_1 = 1.20$, $a_2 = -0.317$

Decay	Br	Decay	Br	Decay	Br
$B_c \to B_s e\nu$	0.84	$B_c^+ \to B_s K^{*+}$	0.003	$B_c^+ \to B^{*0} K^{*+}$	0.033
$B_c \to B_s^* e\nu$	1.75	$B_c^+ \to B_s^* K^+$	0.11	$B_c^+ \to B^+ \bar{K}^0$	0.24
$B_c \to B e\nu$	0.042	$B_c^+ \to B^0 \pi^+$	0.10	$B_c^+ \to B^+ \bar{K}^{*0}$	0.09
$B_c \to B^* e\nu$	0.12	$B_c^+ \to B^0 \rho^+$	0.13	$B_c^+ \to B^{*+} \bar{K}^0$	0.11
$B_c^+ \to B_s \pi^+$	2.52	$B_c^+ \to B^{*0} \pi^+$	0.026	$B_c^+ \to B^{*+} \bar{K}^{*0}$	0.84
$B_c^+ \to B_s \rho^+$	1.41	$B_c^+ \to B^{*0} \rho^+$	0.68	$B_c^+ \to B^+ \pi^0$	0.004
$B_c^+ \to B_s^* \pi^+$	1.61	$B_c^+ \to B^0 K^+$	0.009	$B_c^+ \to B^+ \rho^0$	0.005
$B_c^+ \to B_s^* \rho^+$	11.1	$B_c^+ \to B^0 K^{*+}$	0.004	$B_c^+ \to B^{*+} \pi^0$	0.001
$B_c^+ \to B_s K^+$	0.21	$B_c^+ \to B^{*0} K^+$	0.004	$B_c^+ \to B^{*+} \rho^0$	0.024

$\epsilon_q(p)$ $(q = s, d)$ by the center of mass energies E_q on mass shell was performed. It was shown that such substitution introduces only minor errors which are of the same order as the higher order terms in the heavy quark expansion. Thus the decay form factors were evaluated up to the subleading order of the heavy quark expansion. The overall subleading contributions are small and weakly depend on the momentum transfer q^2.

We calculated semileptonic and non-leptonic (in factorization approximation) B_c decay rates. Our predictions for the branching fractions are summarized in Table 5, where we use the central experimental value of the B_c meson lifetime [17]. From this table we see that the considered semileptonic decays to B_s and B mesons give in total $\sim 2.0\%$ of the B_c decay rate, while the energetic non-leptonic decays provide the dominant contribution $\sim 19.3\%$. In our recent paper [3] we calculated weak B_c decays to charmonium and D mesons, associated with $\bar{b} \to \bar{c}, \bar{u}$ quark transition. It was found that the semileptonic decays to the ground and first radially excited states of charmonium and to D mesons yield $\sim 1.7\%$ and corresponding energetic non-leptonic decays (to charmonium and $K^{(*)}$ or π, ρ mesons) contribute $\sim 0.6\%$. All these decays (to B_s, B, charmonium and D mesons) add up to $\sim 23.6\%$ of the B_c total decay rate.

Acknowledgements. The authors express their gratitude to M. Müller-Preussker and V. Savrin for support and discussions. Two of us (R.N.F and V.O.G.) were supported in part by the Deutsche Forschungsgemeinschaft under contract Eb 139/2-2.

A Form factors of weak B_c decays

(a) $B_c \to P$ transitions $(P = B_s, B)$

$$f_+^{(1)}(q^2) = \sqrt{\frac{E_P}{M_{B_c}}} \int \frac{d^3 p}{(2\pi)^3} \bar{\Psi}_P \left(\mathbf{p} + \frac{2m_b}{E_P + M_P}\boldsymbol{\Delta}\right) \times \sqrt{\frac{\epsilon_q(p+\Delta) + m_q}{2\epsilon_q(p+\Delta)}} \sqrt{\frac{\epsilon_c(p) + m_c}{2\epsilon_c(p)}} \times \left\{1 + \frac{M_{B_c} - E_P}{\epsilon_q(p+\Delta) + m_q} + \frac{(\mathbf{p}\boldsymbol{\Delta})}{\boldsymbol{\Delta}^2} \left(\frac{\boldsymbol{\Delta}^2}{[\epsilon_q(p+\Delta) + m_q][\epsilon_c(p) + m_c]} + (M_{B_c} - E_P)\left(\frac{1}{\epsilon_q(p+\Delta) + m_q} + \frac{1}{\epsilon_c(p) + m_c}\right)\right) + \frac{\mathbf{p}^2}{[\epsilon_q(p+\Delta) + m_q][\epsilon_c(p) + m_c]} + \frac{2}{3}\mathbf{p}^2 \left(\frac{E_P - M_P}{[\epsilon_q(p+\Delta) + m_q][\epsilon_c(p) + m_c]} \times \left(\frac{1}{\epsilon_q(p+\Delta) + m_q} - \frac{1}{\epsilon_b(p) + m_b}\right) + \frac{M_{B_c} - E_P}{E_P + M_P}\left(\frac{1}{\epsilon_q(p+\Delta) + m_q} - \frac{1}{\epsilon_c(p) + m_c}\right) \times \left(\frac{1}{\epsilon_b(p) + m_b} - \frac{1}{\epsilon_q(p+\Delta) + m_q}\right)\right)\right\} \Psi_{B_c}(\mathbf{p}), \tag{38}$$

$$f_+^{S(2)}(q^2) = \sqrt{\frac{E_P}{M_{B_c}}} \int \frac{d^3 p}{(2\pi)^3} \bar{\Psi}_P \left(\mathbf{p} + \frac{2m_b}{E_P + M_P}\boldsymbol{\Delta}\right) \sqrt{\frac{E_q + m_q}{2E_q}} \times \left\{\frac{1}{E_q}\left[\frac{E_q - m_q}{E_q + m_q}\left(1 - \frac{E_q + m_q}{2m_c}\right) - \frac{M_{B_c} - E_P}{E_q + mq}\right] \times \left[M_P - \epsilon_q\left(p + \frac{2m_b}{E_P + M_P}\Delta\right) - \epsilon_b\left(p + \frac{2m_b}{E_P + M_P}\Delta\right)\right] \right. \tag{39}$$

$$+ \frac{(\mathbf{p}\boldsymbol{\Delta})}{\boldsymbol{\Delta}^2}$$

$$\times\left(\frac{1}{2E_q}\left[\frac{\mathbf{\Delta}^2}{(E_q+m_q)^2}-\frac{M_{B_c}-E_P}{E_q+m_q}\left(1+\frac{E_q-m_q}{2m_c}\right)\right]\right.$$
$$\times\Bigg[M_{B_c}+M_P-\epsilon_b(p)-\epsilon_c(p)$$
$$-\epsilon_q\left(p+\frac{2m_b}{E_P+M_P}\Delta\right)-\epsilon_b\left(p+\frac{2m_b}{E_P+M_P}\Delta\right)\Bigg]$$
$$-\frac{\mathbf{\Delta}^2}{2m_cE_q(E_q+m_q)}\left[M_P-\epsilon_q\left(p+\frac{2m_b}{E_P+M_P}\Delta\right)\right.$$
$$\left.\left.\left.-\ \epsilon_b\left(p+\frac{2m_b}{E_P+M_P}\Delta\right)\right]\right)\right\}\Psi_{B_c}(\mathbf{p}),$$

$$f_+^{V(2)}(q^2)$$
$$=\sqrt{\frac{E_P}{M_{B_c}}}\int\frac{d^3p}{(2\pi)^3}\bar{\Psi}_P\left(\mathbf{p}+\frac{2m_b}{E_P+M_P}\mathbf{\Delta}\right)\sqrt{\frac{E_q+m_q}{2E_q}}$$
$$\times\left\{-\frac{(E_q-m_q)(m_Q+M_{B_c}-E_P)}{2m_bE_q(E_q+m_q)}\right. \tag{40}$$
$$\times\Bigg[M_{B_c}+M_P-\epsilon_b(p)-\epsilon_c(p)$$
$$-\epsilon_q\left(p+\frac{2m_b}{E_P+M_P}\Delta\right)-\epsilon_b\left(p+\frac{2m_b}{E_P+M_P}\Delta\right)\Bigg]$$
$$+\frac{(\mathbf{p\Delta})}{\mathbf{\Delta}^2}\left(\frac{E_q-m_q}{2E_qm_q}\left(\frac{E_q-m_q}{E_q+m_q}\left[1-\frac{E_q+m_q}{2m_c}\right]\right.\right.$$
$$\times[M_{B_c}-\epsilon_b(p)-\epsilon_c(p)]$$
$$-\left[1-\frac{E_q-m_q}{2m_c}\right]\left[M_P-\epsilon_q\left(p+\frac{2m_b}{E_P+M_P}\Delta\right)\right.$$
$$\left.\left.\left.-\epsilon_b\left(p+\frac{2m_b}{E_P+M_P}\Delta\right)\right]\right)\right)$$
$$-\frac{1}{2m_b}\left[\frac{m_q}{E_q}\left(\frac{E_q-m_q}{E_q+m_q}-\frac{M_{B_c}-E_P}{E_q+m_q}\right)\right.$$
$$\left.+\ \frac{E_q-m_q}{E_q}\frac{M_{B_c}-E_P}{E_q+m_q}\right]$$
$$\times\Bigg[M_{B_c}+M_P-\epsilon_b(p)-\epsilon_c(p)$$
$$-\epsilon_q\left(p+\frac{2m_b}{E_P+M_P}\Delta\right)$$
$$\left.\left.\left.-\epsilon_b\left(p+\frac{2m_b}{E_P+M_P}\Delta\right)\right]\right)\right\}\Psi_{B_c}(\mathbf{p}),$$

$$f_0^{(1)}(q^2)$$
$$=\frac{2\sqrt{E_PM_{B_c}}}{M_{B_c}^2-M_P^2}\int\frac{d^3p}{(2\pi)^3}\bar{\Psi}_P\left(\mathbf{p}+\frac{2m_b}{E_P+M_P}\mathbf{\Delta}\right)$$
$$\times\sqrt{\frac{\epsilon_q(p+\Delta)+m_q}{2\epsilon_q(p+\Delta)}}\sqrt{\frac{\epsilon_c(p)+m_c}{2\epsilon_c(p)}} \tag{41}$$
$$\times\left\{(E_P+M_P)\left(\frac{E_P-M_P}{\epsilon_q(p+\Delta)+m_q}+\frac{M_{B_c}-E_P}{E_P+M_P}\right)\right.$$
$$+(\mathbf{p\Delta})\left(\frac{1}{\epsilon_q(p+\Delta)+m_q}+\frac{1}{\epsilon_c(p)+m_c}\right.$$
$$\left.+\ \frac{M_{B_c}-E_P}{[\epsilon_q(p+\Delta)+m_q][\epsilon_c(p)+m_c]}\right)$$
$$+\frac{\mathbf{p}^2(M_{B_c}-E_P)}{[\epsilon_q(p+\Delta)+m_q][\epsilon_c(p)+m_c]}$$
$$-\frac{2}{3}\mathbf{p}^2(E_P-M_P)$$
$$\times\left(\frac{1}{\epsilon_q(p+\Delta)+m_q}-\frac{1}{\epsilon_b(p)+m_b}\right)$$
$$\times\left(\frac{1}{\epsilon_q(p+\Delta)+m_q}-\frac{1}{\epsilon_c(p)+m_c}\right.$$
$$\left.\left.-\ \frac{M_{B_c}-E_P}{[\epsilon_q(p+\Delta)+m_q][\epsilon_c(p)+m_c]}\right)\right\}\Psi_{B_c}(\mathbf{p}),$$

$$f_0^{S(2)}(q^2)$$
$$=\frac{2\sqrt{E_PM_{B_c}}}{M_{B_c}^2-M_P^2}\int\frac{d^3p}{(2\pi)^3}\bar{\Psi}_P\left(\mathbf{p}+\frac{2m_b}{E_P+M_P}\mathbf{\Delta}\right)$$
$$\times\sqrt{\frac{E_q+m_q}{2E_q}}\frac{1}{E_q(E_q+m_q)} \tag{42}$$
$$\times\left\{\left[(E_q-m_q)(M_{B_c}-E_P)\left(1-\frac{E_q+m_q}{2m_c}\right)-\mathbf{\Delta}^2\right]\right.$$
$$\times\left[M_P-\epsilon_q\left(p+\frac{2m_b}{E_P+M_P}\Delta\right)\right.$$
$$\left.-\epsilon_b\left(p+\frac{2m_b}{E_P+M_P}\Delta\right)\right]$$
$$+\frac{(\mathbf{p\Delta})}{2}\left(\left[\frac{M_{B_c}-E_P}{E_q+m_q}-1-\frac{E_q-m_q}{2m_c}\right]\right.$$
$$\times\Bigg[M_{B_c}+M_P-\epsilon_b(p)-\epsilon_c(p)$$
$$-\epsilon_q\left(p+\frac{2m_b}{E_P+M_P}\Delta\right)-\epsilon_b\left(p+\frac{2m_b}{E_P+M_P}\Delta\right)\Bigg]$$
$$-\frac{1}{m_c}[M_{B_c}-E_P-2(E_q-m_q)]$$
$$\times\left[M_P-\epsilon_q\left(p+\frac{2m_b}{E_P+M_P}\Delta\right)\right.$$
$$\left.\left.\left.-\ \epsilon_b\left(p+\frac{2m_b}{E_P+M_P}\Delta\right)\right]\right)\right\}\Psi_{B_c}(\mathbf{p}),$$

$$f_0^{V(2)}(q^2) = \frac{2\sqrt{E_P M_{B_c}}}{M_{B_c}^2 - M_P^2}\int \frac{d^3p}{(2\pi)^3}\bar{\Psi}_P\left(\mathbf{p} + \frac{2m_b}{E_P+M_P}\mathbf{\Delta}\right) \times\sqrt{\frac{E_q+m_q}{2E_q}}\frac{1}{2E_q(E_q+m_q)} \times\left\{-\frac{E_q-m_q}{m_b}[m_q(M_{B_c}-E_{B_c})+\mathbf{\Delta}^2] \times\left[M_{B_c}+M_P-\epsilon_b(p)-\epsilon_c(p) -\epsilon_q\left(p+\frac{2m_b}{E_P+M_P}\Delta\right)-\epsilon_b\left(p+\frac{2m_b}{E_P+M_P}\Delta\right)\right] +(\mathbf{p\Delta})\left(\frac{M_{B_c}-E_{B_c}}{m_q}\left(\frac{E_q-m_q}{E_q+m_q}\left[1-\frac{E_q+m_q}{2m_c}\right] \times[M_{B_c}-\epsilon_b(p)-\epsilon_c(p)]-\left[1-\frac{E_q-m_q}{2m_c}\right] \times\left[M_P-\epsilon_q\left(p+\frac{2m_b}{E_P+M_P}\Delta\right) -\epsilon_b\left(p+\frac{2m_b}{E_P+M_P}\Delta\right)\right]\right) -\frac{1}{m_b}\left[\frac{m_q}{E_q+m_q}(M_{B_c}-E_{B_c}-E_q-m_q)+E_q-m_q\right] \times\left[M_{B_c}+M_P-\epsilon_b(p)-\epsilon_c(p) -\epsilon_q\left(p+\frac{2m_b}{E_P+M_P}\Delta\right) -\epsilon_b\left(p+\frac{2m_b}{E_P+M_P}\Delta\right)\right]\right)\right\}\Psi_{B_c}(\mathbf{p}), \tag{43}$$

where

$$|\mathbf{\Delta}| = \sqrt{\frac{(M_{B_c}^2+M_P^2-q^2)^2}{4M_{B_c}^2} - M_P^2},$$

$$E_P = \sqrt{M_P^2+\mathbf{\Delta}^2}, \quad \epsilon_Q(p+\lambda\Delta) = \sqrt{m_Q^2+(\mathbf{p}+\lambda\mathbf{\Delta})^2} \quad (Q=b,s,d),$$

and the subscript q corresponds to s or d quark for the final B_s or B meson, respectively.
(b) $B_c \to V$ transitions ($V = B_s^*, B^*$)

$$V^{(1)}(q^2) = \frac{M_{B_c}+M_V}{2\sqrt{M_{B_c}M_V}}\int\frac{d^3p}{(2\pi)^3}\bar{\Psi}_V\left(\mathbf{p}+\frac{2m_b}{E_V+M_V}\mathbf{\Delta}\right) \times\sqrt{\frac{\epsilon_q(p+\Delta)+m_q}{2\epsilon_q(p+\Delta)}}\sqrt{\frac{\epsilon_c(p)+m_c}{2\epsilon_c(p)}} \times\frac{2\sqrt{E_V M_V}}{\epsilon_q(p+\Delta)+m_q}\left\{1+\frac{(\mathbf{p\Delta})}{\mathbf{\Delta}^2}\left(1-\frac{\epsilon_q(p+\Delta)+m_q}{2m_c}\right) +\frac{2}{3}\frac{\mathbf{p}^2}{E_V+M_V} \times\left(\frac{\epsilon_q(p+\Delta)+m_q}{2m_c[\epsilon_b(p)+m_b]}-\frac{1}{\epsilon_q(p+\Delta)+m_q}\right)\right\}\Psi_{B_c}(\mathbf{p}), \tag{44}$$

$$V^{S(2)}(q^2) = \frac{M_{B_c}+M_V}{2\sqrt{M_{B_c}M_V}}\int\frac{d^3p}{(2\pi)^3}\bar{\Psi}_V\left(\mathbf{p}+\frac{2m_b}{E_V+M_V}\mathbf{\Delta}\right) \times\sqrt{\frac{E_q+m_q}{2E_q}}\frac{2\sqrt{E_V M_V}}{E_q+m_q} \times\left\{-\frac{1}{E_q}\left(1+\frac{E_q-m_q}{4m_c}\right) \times\left[M_V-\epsilon_q\left(p+\frac{2m_b}{E_V+M_V}\Delta\right) -\epsilon_b\left(p+\frac{2m_b}{E_V+M_V}\Delta\right)\right] -\frac{(\mathbf{p\Delta})}{\mathbf{\Delta}^2}\left(\frac{1}{2E_q}\left[M_{B_c}+M_V-\epsilon_b(p)-\epsilon_c(p) -\epsilon_q\left(p+\frac{2m_b}{E_V+M_V}\Delta\right)-\epsilon_b\left(p+\frac{2m_b}{E_V+M_V}\Delta\right)\right] +\frac{E_q-m_q}{2m_cE_q}\left[M_V-\epsilon_q\left(p+\frac{2m_b}{E_V+M_V}\Delta\right) -\epsilon_b\left(p+\frac{2m_b}{E_V+M_V}\Delta\right)\right]\right)\right\}\Psi_{B_c}(\mathbf{p}), \tag{45}$$

$$V^{V(2)}(q^2) = \frac{M_{B_c}+M_V}{2\sqrt{M_{B_c}M_V}}\int\frac{d^3p}{(2\pi)^3}\bar{\Psi}_V\left(\mathbf{p}+\frac{2m_b}{E_V+M_V}\mathbf{\Delta}\right) \times\sqrt{\frac{E_q+m_q}{2E_q}}\frac{2\sqrt{E_V M_V}}{E_q+m_q}\left\{\frac{E_q-m_q}{4E_qm_c} \times\left[M_V-\epsilon_q\left(p+\frac{2m_b}{E_V+M_V}\Delta\right) -\epsilon_b\left(p+\frac{2m_b}{E_V+M_V}\Delta\right)\right] -\frac{(\mathbf{p\Delta})}{\mathbf{\Delta}^2}\left(\frac{E_q-m_q}{4E_qm_q}\left(1+\frac{E_q-m_q}{2m_c}\right) \times\left[M_{B_c}-M_V-\epsilon_b(p)-\epsilon_c(p) +\epsilon_q\left(p+\frac{2m_b}{E_V+M_V}\Delta\right)+\epsilon_b\left(p+\frac{2m_b}{E_V+M_V}\Delta\right)\right]\right.\right. \tag{46}$$

$$
-\frac{E_q+m_q}{4E_q m_b}\left[M_{B_c}+M_V-\epsilon_b(p)-\epsilon_c(p)-\epsilon_q\left(p+\frac{2m_b}{E_V+M_V}\Delta\right)-\epsilon_b\left(p+\frac{2m_b}{E_V+M_V}\Delta\right)\right]\Bigg)\Bigg\}\Psi_{B_c}(\mathbf{p}),
$$

$$
A_1^{(1)}(q^2)=\frac{2\sqrt{M_{B_c}M_V}}{M_{B_c}+M_V}\sqrt{\frac{E_V}{M_V}}\int\frac{\mathrm{d}^3p}{(2\pi)^3}\bar\Psi_V\left(\mathbf{p}+\frac{2m_b}{E_V+M_V}\mathbf{\Delta}\right)\times\sqrt{\frac{\epsilon_q(p+\Delta)+m_q}{2\epsilon_q(p+\Delta)}}\sqrt{\frac{\epsilon_c(p)+m_c}{2\epsilon_c(p)}}\times\left\{1+\frac{1}{2m_c[\epsilon_q(p+\Delta)+m_q]}\times\left[\frac{2}{3}\mathbf{p}^2\frac{E_V-M_V}{\epsilon_b(p)+m_b}-\frac{\mathbf{p}^2}{3}-(\mathbf{p\Delta})\right]\right\}\Psi_{B_c}(\mathbf{p}), \tag{47}
$$

$$
A_1^{S(2)}(q^2)=\frac{2\sqrt{M_{B_c}M_V}}{M_{B_c}+M_V}\sqrt{\frac{E_V}{M_V}}\int\frac{\mathrm{d}^3p}{(2\pi)^3}\bar\Psi_V\left(\mathbf{p}+\frac{2m_b}{E_V+M_V}\mathbf{\Delta}\right)\times\sqrt{\frac{E_q+m_q}{2E_q}}\frac{E_q-m_q}{E_q(E_q+m_q)}\left(1+\frac{E_q-m_q}{2m_c}\right)\times\left[M_V-\epsilon_q\left(p+\frac{2m_b}{E_V+M_V}\Delta\right)-\epsilon_b\left(p+\frac{2m_b}{E_V+M_V}\Delta\right)\right]\Psi_{B_c}(\mathbf{p}), \tag{48}
$$

$$
A_1^{V(2)}(q^2)=\frac{2\sqrt{M_{B_c}M_V}}{M_{B_c}+M_V}\sqrt{\frac{E_V}{M_V}}\int\frac{\mathrm{d}^3p}{(2\pi)^3}\bar\Psi_V\left(\mathbf{p}+\frac{2m_b}{E_V+M_V}\mathbf{\Delta}\right)\times\sqrt{\frac{E_q+m_q}{2E_q}}\,\frac{E_q-m_q}{2E_q(E_q+m_q)}\times\frac{(\mathbf{p\Delta})}{\mathbf{\Delta}^2}\Bigg\{-\left(1+\frac{m_q}{m_b}\right)\times\left[M_{B_c}+M_V-\epsilon_b(p)-\epsilon_c(p)-\epsilon_q\left(p+\frac{2m_b}{E_V+M_V}\Delta\right)-\epsilon_b\left(p+\frac{2m_b}{E_V+M_V}\Delta\right)\right]+\frac{E_q}{m_q}\left(1+\frac{E_q^2-m_q^2}{2E_q m_c}\right)\left[M_{B_c}-M_V-\epsilon_b(p)-\epsilon_c(p)+\epsilon_q\left(p+\frac{2m_b}{E_V+M_V}\Delta\right)+\epsilon_b\left(p+\frac{2m_b}{E_V+M_V}\Delta\right)\right]\Bigg\}\Psi_{B_c}(\mathbf{p}), \tag{49}
$$

$$
A_2^{(1)}(q^2)=\frac{M_{B_c}+M_V}{2\sqrt{M_{B_c}M_V}}\frac{2\sqrt{E_V M_V}}{E_V+M_V}\int\frac{\mathrm{d}^3p}{(2\pi)^3}\bar\Psi_V\left(\mathbf{p}+\frac{2m_b}{E_V+M_V}\mathbf{\Delta}\right)\times\sqrt{\frac{\epsilon_q(p+\Delta)+m_q}{2\epsilon_q(p+\Delta)}}\times\sqrt{\frac{\epsilon_c(p)+m_c}{2\epsilon_c(p)}}\Bigg\{1+\frac{M_V}{M_{B_c}}\left(1-\frac{E_V+M_V}{\epsilon_q(p+\Delta)+m_q}\right)-\frac{(\mathbf{p\Delta})}{\mathbf{\Delta}^2}\frac{E_V+M_V}{\epsilon_q(p+\Delta)+m_q}\left(\frac{E_V+M_V}{2m_c}\times\left[1-\frac{M_V}{M_{B_c}}\left(1-\frac{\epsilon_q(p+\Delta)+m_q}{E_V+M_V}\right)\right]+\frac{M_V}{M_{B_c}}\right)+\frac{2}{3}\frac{\mathbf{p}^2}{\epsilon_q(p+\Delta)+m_q}\times\Bigg(\frac{1}{2m_c}\left[\frac{E_V+M_V}{\epsilon_b(p)+m_b}-\frac{1}{2}+\frac{M_V}{\epsilon_q(p+\Delta)+m_q}+\frac{M_V}{M_{B_c}}\left(\frac{\epsilon_q(p+\Delta)+m_q}{\epsilon_b(p)+m_b}-\frac{E_V+M_V}{\epsilon_b(p)+m_b}+\frac{1}{2}-\frac{E_V}{\epsilon_q(p+\Delta)+m_q}\right)\right]+\frac{M_V}{M_{B_c}}\left(\frac{1}{\epsilon_q(p+\Delta)+m_q}+\frac{1}{\epsilon_b(p)+m_b}\right)\Bigg)\Bigg\}\Psi_{B_c}(\mathbf{p}), \tag{50}
$$

$$
A_2^{S(2)}(q^2)=\frac{M_{B_c}+M_V}{2\sqrt{M_{B_c}M_V}}\frac{2\sqrt{E_V M_V}}{E_V+M_V}\int\frac{\mathrm{d}^3p}{(2\pi)^3}\bar\Psi_V\left(\mathbf{p}+\frac{2m_b}{E_V+M_V}\mathbf{\Delta}\right)\times\sqrt{\frac{E_q+m_q}{2E_q}}\Bigg\{\frac{E_q-m_q}{E_q(E_q+m_q)}\left[1+\frac{E_q-m_q}{2m_c}+\frac{M_V}{M_{B_c}}\left(1+\frac{E_V+M_V}{E_q-m_q}+\frac{E_q-m_q}{2m_c}\right)\right]\times\left[M_V-\epsilon_q\left(p+\frac{2m_b}{E_V+M_V}\Delta\right)-\epsilon_b\left(p+\frac{2m_b}{E_V+M_V}\Delta\right)\right]-\frac{(\mathbf{p\Delta})}{\mathbf{\Delta}^2}\frac{M_V}{M_{B_c}}\frac{E_V+M_V}{E_q(E_q+m_q)} \tag{51}
$$

$$\times\left(\frac{1}{2}\left(1-\frac{E_q-m_q}{2m_c}\right)\left[M_{B_c}+M_V-\epsilon_b(p)-\epsilon_c(p)\right.\right.$$
$$-\epsilon_q\left(p+\frac{2m_b}{E_V+M_V}\Delta\right)-\epsilon_b\left.\left(p+\frac{2m_b}{E_V+M_V}\Delta\right)\right]$$
$$-\frac{M_{B_c}-E_V}{m_c}\left[M_V-\epsilon_q\left(p+\frac{2m_b}{E_V+M_V}\Delta\right)\right.$$
$$\left.\left.\left.-\epsilon_b\left(p+\frac{2m_b}{E_V+M_V}\Delta\right)\right]\right)\right\}\Psi_{B_c}(\mathbf{p}),$$

$$A_2^{V(2)}(q^2)$$
$$=\frac{M_{B_c}+M_V}{2\sqrt{M_{B_c}M_V}}\frac{2\sqrt{E_VM_V}}{E_V+M_V}\int\frac{d^3p}{(2\pi)^3}\bar{\Psi}_V\left(\mathbf{p}+\frac{2m_b}{E_V+M_V}\mathbf{\Delta}\right)$$
$$\times\sqrt{\frac{E_q+m_q}{2E_q}}\frac{(\mathbf{p\Delta})}{\mathbf{\Delta}^2}\frac{E_q-m_q}{2E_q(E_q+m_q)} \quad (52)$$
$$\times\left\{-\left(1+\frac{m_q}{m_b}+\frac{M_V}{M_{B_c}}\left[1+\frac{m_q}{m_b}\left(1+\frac{E_V+M_V}{E_q-m_q}\right)\right]\right)\right.$$
$$\times\left[M_{B_c}+M_V-\epsilon_b(p)-\epsilon_c(p)-\epsilon_q\left(p+\frac{2m_b}{E_V+M_V}\Delta\right)\right.$$
$$\left.-\epsilon_b\left(p+\frac{2m_b}{E_V+M_V}\Delta\right)\right]$$
$$+\frac{E_q}{m_q}\left[1+\frac{E_q^2-m_q^2}{2E_qm_q}\right.$$
$$\left.+\frac{M_V}{M_{B_c}}\left(1+\frac{E_V+M_V}{E_q}+\frac{E_q^2-m_q^2}{2E_qm_q}\right)\right]$$
$$\times\left[M_{B_c}-M_V-\epsilon_b(p)-\epsilon_c(p)+\epsilon_q\left(p+\frac{2m_b}{E_V+M_V}\Delta\right)\right.$$
$$\left.\left.+\epsilon_b\left(p+\frac{2m_b}{E_V+M_V}\Delta\right)\right]\right\}\Psi_{B_c}(\mathbf{p}),$$

$$A_0^{(1)}(q^2)$$
$$=\sqrt{\frac{E_V}{M_V}}\int\frac{d^3p}{(2\pi)^3}\bar{\Psi}_V\left(\mathbf{p}+\frac{2m_b}{E_V+M_V}\mathbf{\Delta}\right)$$
$$\times\sqrt{\frac{\epsilon_q(p+\Delta)+m_q}{2\epsilon_q(p+\Delta)}}\sqrt{\frac{\epsilon_c(p)+m_c}{2\epsilon_c(p)}} \quad (53)$$
$$\times\left\{1+\frac{M_{B_c}-E_V}{\epsilon_q(p+\Delta)+m_q}\right.$$
$$\times\left(1+\left[\frac{(\mathbf{p\Delta})}{\mathbf{\Delta}^2}-\frac{2}{3}\frac{\mathbf{p}^2}{E_V+M_V}\right.\right.$$
$$\left.\times\left(\frac{1}{\epsilon_q(p+\Delta)+m_q}+\frac{1}{\epsilon_b(p)+m_b}\right)\right]$$
$$\left.\times\left[1+\frac{1}{2m_c}\left(\frac{\mathbf{\Delta}^2}{M_{B_c}-E_V}+\epsilon_q(p+\Delta)+m_q\right)\right]\right)$$
$$\left.-\frac{\mathbf{p}^2}{6m_c[\epsilon_q(p+\Delta)+m_q]}\right\}\Psi_{B_c}(\mathbf{p}),$$

$$A_0^{S(2)}(q^2)$$
$$=\sqrt{\frac{E_V}{M_V}}\int\frac{d^3p}{(2\pi)^3}\bar{\Psi}_V\left(\mathbf{p}+\frac{2m_b}{E_V+M_V}\mathbf{\Delta}\right)\sqrt{\frac{E_q+m_q}{2E_q}}$$
$$\times\frac{1}{E_q(E_q+m_q)}$$
$$\times\left\{\left[(E_q-m_q)\left(1-\frac{E_q-m_q}{2m_c}\right)-M_{B_c}+E_V\right]\right. \quad (54)$$
$$\times\left[M_V-\epsilon_q\left(p+\frac{2m_b}{E_V+M_V}\Delta\right)\right.$$
$$\left.-\epsilon_b\left(p+\frac{2m_b}{E_V+M_V}\Delta\right)\right]$$
$$-\frac{(\mathbf{p\Delta})}{\mathbf{\Delta}^2}\left(\frac{M_{B_c}-E_V}{2}\left(1-\frac{E_q-m_q}{2m_c}\right)\right.$$
$$\times\left[M_{B_c}+M_V-\epsilon_b(p)-\epsilon_c(p)-\epsilon_q\left(p+\frac{2m_b}{E_V+M_V}\Delta\right)\right.$$
$$\left.-\epsilon_b\left(p+\frac{2m_b}{E_V+M_V}\Delta\right)\right]$$
$$+\frac{\mathbf{\Delta}^2}{m_c}\left[M_V-\epsilon_q\left(p+\frac{2m_b}{E_V+M_V}\Delta\right)\right.$$
$$\left.\left.\left.-\epsilon_b\left(p+\frac{2m_b}{E_V+M_V}\Delta\right)\right]\right)\right\}\Psi_{B_c}(\mathbf{p}),$$

$$A_0^{V(2)}(q^2)$$
$$=\sqrt{\frac{E_V}{M_V}}\int\frac{d^3p}{(2\pi)^3}\bar{\Psi}_V\left(\mathbf{p}+\frac{2m_b}{E_V+M_V}\mathbf{\Delta}\right)\sqrt{\frac{E_q+m_q}{2E_q}}$$
$$\times\frac{(\mathbf{p\Delta})}{\mathbf{\Delta}^2}\frac{1}{2E_q(E_q+m_q)} \quad (55)$$
$$\times\left\{\frac{1}{m_q}\left[\left(\frac{E_q-m_q}{2m_c}+\frac{E_q}{E_q+m_q}\right)\mathbf{\Delta}^2\right.\right.$$
$$\left.-(E_q-m_q)(M_{B_c}-E_V)\right]$$
$$\times\left[M_{B_c}-M_V-\epsilon_b(p)-\epsilon_c(p)+\epsilon_q\left(p+\frac{2m_b}{E_V+M_V}\Delta\right)\right.$$
$$\left.+\epsilon_b\left(p+\frac{2m_b}{E_V+M_V}\Delta\right)\right]$$
$$-\left[\frac{\mathbf{\Delta}^2}{E_q+m_q}+\frac{m_q}{m_b}\left(\frac{\mathbf{\Delta}^2}{E_q+m_q}-M_{B_c}+E_V\right)\right]$$

$$\times \Bigg[M_{B_c} + M_V - \epsilon_b(p) - \epsilon_c(p) - \epsilon_q\left(p + \frac{2m_b}{E_V + M_V}\Delta\right) - \epsilon_b\left(p + \frac{2m_b}{E_V + M_V}\Delta\right)\Bigg]\Bigg\} \Psi_{B_c}(\mathbf{p}),$$

where

$$|\mathbf{\Delta}| = \sqrt{\frac{(M_{B_c}^2 + M_V^2 - q^2)^2}{4M_{B_c}^2} - M_V^2},$$

$$E_V = \sqrt{M_V^2 + \mathbf{\Delta}^2}.$$

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